MEASUREMENT OF TRANSIENT HEAT FLUX USING HEAT GAUGES

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This paper describes the theoretical basis, the design, and the method of calibrating heat gauges for measuring variable heat fluxes.

Auxiliary Wall Type of Heat Gauge*

Heat gauges based on the so-called auxiliary wall technique are widely used to measure heat flux. These gauges take the form of a thermal insulator carrying a differential thermocouple or thermopile. Recently, semiconductor heat gauges have found increasing use. For a steady flux P the relation between the flux and the potential difference ΔE or temperature difference Δt_m on the surface of a heat gauge is given by the well-known formula [1]

$$P = g\Delta E = k\Delta t_m. \tag{1}$$

We consider the possible measurement of an unsteady heat flux using this type of gauge. We represent the gauge in the form of an infinite flat plate, one surface of which absorbs a heat flux P(τ), while the second surface exchanges heat with surrounding space in a different manner, i.e., there are several possible conditions at the boundary x = l (Fig. 1a). Assuming that the thermophysical properties of the heat gauge and the heat-transfer conditions at the boundaries are independent of temperature, we can write a differential equation for the temperature field in the plate:

$$\lambda \frac{\partial^2 t}{\partial x^2} = c\rho \frac{\partial t}{\partial \tau}. \tag{2}$$

We now apply the operator

$$I[f] = \frac{1}{l}\int_0^l f dx \tag{3}$$

to both sides of the equation, apply the boundary conditions at x = 0:

$$P(\tau) = -\lambda \frac{\partial t}{\partial x}\Big|_{x=0} S \tag{4}$$

and transform Eq. (2)

$$\frac{\lambda}{l}\int_0^l \frac{\partial^2 t}{\partial x^2} dx = \frac{\lambda}{l}\int_0^l \partial\left(\frac{\partial t}{\partial x}\right) = \frac{1}{l}\left[\lambda \frac{\partial t}{\partial x}\Big|_{x=l} - \lambda_{\text{T}} \frac{\partial t}{\partial x}\Big|_{x=0}\right],$$

$$\frac{c\rho}{l}\int_0^l \frac{\partial t}{\partial \tau} dx = c\rho \frac{\partial t_v}{\partial \tau}.$$

Here t_v denotes the volume average temperature,

*We shall designate these heat gauges as "ordinary" in the remainder of the paper.

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$$I[t] = \frac{1}{l}\int_0^l t dx = t_v. \tag{5}$$

Following transformation Eq. (2) takes the form

$$P(\tau) = C\,\frac{dt_v}{d\tau} - \lambda S\,\frac{\partial t}{\partial x}\bigg|_{x=l}. \tag{6}$$

The last term in Eq. (6) can be expressed through the condition at the boundary $x = l$. The surface $x = l$ can dissipate heat into a vacuum, or to a gaseous or liquid medium, or it can be attached to the surface of a solid body. In other words, there can be boundary conditions of the third or fourth kind, i.e.,

$$-\lambda\,\frac{\partial t}{\partial x}\bigg|_{x=l} = \alpha(t_l - t_m), \quad -\lambda\,\frac{\partial t}{\partial x}\bigg|_{x=l} = -\lambda_T\,\frac{\partial t_T}{\partial x}\bigg|_{x=l}. \tag{7}$$

Equation (6) indicates that it is possible to measure a heat flux $P(\tau)$ which varies with time, using heat gauges of this type. To do this one must know the variation of the volume average temperature t_v with time and the temperature gradient at the boundary $x = l$ at different times.

We consider possible measurement of these quantities. Measurement of the average volume temperature of a gauge encounters more or less difficulty depending on the gauge structure, and, as a rule, requires some modification, since the heat gauge is constructed for possible measurement of temperature drop at its surface.

Measurement of the temperature gradient at the surface $x = l$ is rather more complicated. One cannot measure this directly, but can only do so with the help of conditions (7). The following relations can be derived from Eqs. (6) and (7):

$$P(\tau) = C\,\frac{dt_v}{d\tau} + \alpha S(t_l - t_c), \tag{8}$$

$$P(\tau) = C\,\frac{dt_v}{d\tau} - \lambda_T\,\frac{\partial t_T}{\partial x}\bigg|_{x=l}. \tag{9}$$

It follows from Eqs. (8) and (9) that to determine the flux $P(\tau)$, besides the variation of average volume temperature with time, one must measure either the temperature at the wall $x = l$ and the temperature t_m of the surrounding medium, or the temperature gradient $\partial t_T/\partial x|_{x=l}$ in the body to which the gauge is attached. In addition, one must know the parameters α and λ_T which describe the gauge operating conditions.

Although it is possible to measure the gradient $\partial t_T/\partial x|_{x=l}$, there are major engineering difficulties. As regards the parameters α and λ_T, these can be determined from calibration tests, but such tests are valid only for the conditions in which the test data are obtained.

Thus, analysis of Eq. (6) leads to the following conclusions:

The measurement of a heat flux $P(\tau)$ which varies with time cannot be made using formulas of type (1), obtained for steady conditions;

ordinary heat gauges based on the auxiliary wall method are not well suited for the unsteady problem;

the greatest difficulty in the measurement stems from determination of the last term in Eqs. (6), (8), and (9).

Combination and RC Heat Gauges

We consider a system of bodies consisting of an ordinary heat gauge 1 and a wall 2 separated from 1 by a gas gap 3 (Fig. 1b). In practice, this system can be implemented using a cap 2, attached to the gauge 1 and separated from it by a narrow gap 3 and insulating gasket 4 (Fig. 1c). We assume that we can measure the temperature difference $\Delta t = t_1|_{x=l_1} - t_2|_{x=l_2}$, and then the last term in Eq. (6) can be put in the form

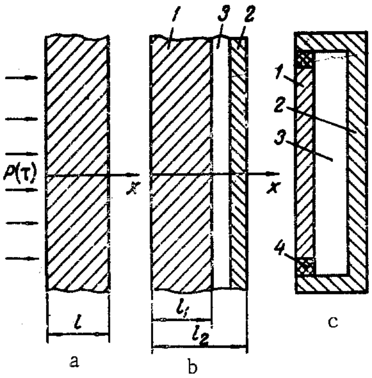

Fig. 1. Heat gauge models.

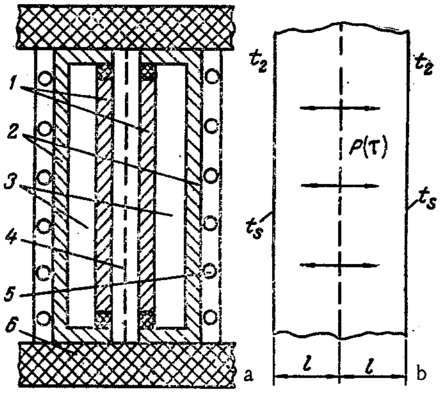

Fig. 2. Schematic of test rig for heat gauge calibration.

$$-\lambda S \frac{\partial t}{\partial x}\bigg|_{x=l} = \sigma \Delta t,$$

and Eq. (6) takes the form

$$P(\tau) = C \frac{dt_v}{d\tau} + \sigma \Delta t. \tag{10}$$

Equation (10) assumes that the gap 3 possesses negligibly small heat capacity in comparison with the capacity C of the gauge. The parameter σ, the thermal conductivity of the gap, is independent of the gauge operating conditions. The gap size is chosen so that there is no difficulty in calculating the convective heat flux, which therefore eliminates the conditions making for ambiguity in determining the parameter σ.

The problem of determining P(τ) is appreciably simplified if the temperature distribution in body 1 is uniform, when $t_V = t_1$, and $\Delta t = t_1 - t_2$ ($t_2 \equiv t_2|_{x=l_2}$), and then Eq. (10) takes the form

$$P(\tau) = C \frac{dt_1}{d\tau} + \sigma (t_1 - t_2). \tag{11}$$

Determination of the flux P(τ) from Eq. (11) requires measurement of temperatures t_1 and t_2 of bodies 1 and 2 at different times; the parameters C and σ are constants of the gauge and are found from calibration tests.

Below, we shall call this arrangement an RC gauge, since it represents a body 1 with capacity C and practically zero thermal resistance, combined with body 3 with resistance $R = \sigma^{-1}$, and practically zero heat capacity. We shall call the system of a body with finite resistance and a gap of practically zero capacity a combination heat gauge. Clearly, the RC gauge is a special case of the combination gauge.

We represent Eq. (11) in the form

$$P(\tau) = \frac{1}{m_0 R} \cdot \frac{dt_1}{d\tau} + \frac{1}{R}(t_1 - t_2), \tag{12}$$

$$m_0 = \frac{1}{RC}, \quad R = \frac{1}{\sigma}, \tag{13}$$

and consider a method of determining the parameters m_0 and R from calibration tests. Figure 2a shows two identical heat gauges, with a heater 4 located between them, and the faces of the gauges covered by insulators 6. Attached to plates 2 are slabs 5 through which a temperature-control liquid circulates, to maintain temperature t_2 at any prescribed level. Under steady conditions one measures the heater power P and from Eq. (12) with the condition $dt_1/d\tau = 0$ we have an expression for calculating R:

$$R = \frac{2(t_1 - t_2)}{P}. \tag{14}$$

Here it is assumed that the heat flux P from the heater passes equally through both gauges, and that there are no losses through the edges because of the presence of insulator 6.

To determine the parameter m_0 one can use the following test scheme. Heater 4 is switched on and the variation of the temperature of each of the heat gauges is described by Eq. (12), in which we put T = 0. Then

$$m_0 = -\frac{dt_1}{(t_1 - t_2)\,d\tau} = \frac{d\ln(t_1 - t_2)}{d\tau}. \tag{15}$$

Equation (15) is a definition of the cooling rate of the body, and we can use conventional methods of determining this parameter, such as have been developed in control theory [2].

We now consider the combination heat gauge for which Eq. (10) is valid. We designate the temperature at the surface $x = l_1$ of the gauge as $t_1|_{x=l_1} = t_s$, and we shall consider the temperature $t_2|_{x=l_2} = t_2$ as being that of the medium surrounding the gauge. Taking this notation into account, and also the parameters of Eq. (13), we can rewrite Eq. (10) in the form

$$P(\tau) = \frac{1}{m_1 R}\cdot\frac{d(t_s - t_2)}{d\tau} + \frac{1}{R}(t_s - t_2), \tag{16}$$

$$m_1 = m_0\psi, \quad \psi = \frac{t_s - t_2}{t_v - t_2}. \tag{17}$$

The criterion Ψ describes the level of nonuniformity of the temperature field in the body, illustrated in Fig. 2b. This body is a twin heat gauge or a plate of double thickness, at the center of which there is a plane energy source. The body is located in a medium with constant temperature t_2. We know that under steady heat conditions of the first kind the criterion Ψ is independent of time and the parameter m_1 can be determined experimentally in the same way as for the RC gauge, and appropriate computations were carried out using Eq. (15). Under steady conditions of the second and third kinds the criterion Ψ, as was shown in [3], is also invariant with time, so that we can use Eq. (16) to measure $P(\tau)$. However, the method of determining the parameter m_1 and also the error in measuring $P(\tau)$ when the gauge conditions are transitional requires supplementary investigation. In other words, the possibilities of the combined gauges are limited compared with the RC gauges.

Error in Heat Flux Measurement

The RC heat gauge is used to measure the flux $P(\tau)$ absorbed by surface 1 and conducted through the gap 3 (Fig. 1c). Here the temperature t_2 of surface 2 can vary, not only due to the flux $P(\tau)$, but also because of any other flux absorbed by the wall 2. For example, there can be temperature oscillations of the medium washing the wall 2. We give the name "noise" to variations in wall temperature of heat gauges 1 and 2 arising from causes other than the measured heat flux $P(\tau)$. In [4] it was shown that using a relation of type (12) one could measure $P(\tau)$ in the presence of any form of external noise. However, the nature and magnitude of the noise affect the accuracy of measurement of flux $P(\tau)$. It was shown that the error in determining the power depends on: the thermal conditions of the bodies 1 and 2 of the gauge; the parameters m_1 and R which characterize the gauge construction; the accuracy class of the measuring instrument used.

In [4, 6] results were given of an analysis of the thermal conditions of bodies 1 and 2 with different variations in the heat flux $P(\tau)$ and the external noise. A method is described for choosing the parameters m_1 and R and the measuring instruments, so that the error $\Delta P/P$ in determining the heat flux does not exceed the prescribed values.

Responsiveness and Sensitivity of RC Gauges

The responsiveness of a heat gauge is defined as its capacity to react (to vary its temperature Δt_1) to a minimum change in the flux ΔP occurring in a specific time interval $\Delta\tau$, i.e., the responsiveness is associated with the quantity $dt_1/d\tau$. If we differentiate all the terms of Eq. (11) with respect to time, with t_2 = const, and designated $dt_1/d\tau = T(\tau)$, $dP/d\tau = b$, we obtain the differential equation

$$\frac{dT}{d\tau} + m_0 T = \frac{b}{C}, \tag{18}$$

relating the desired parameter T with the variation in power b.

Assuming the initial conditions and solving Eq. (18) for the case b = const, we obtain, in particular, the following formula for determining the coefficient $\varepsilon = 1/m$ of the thermal responsiveness of the gauge [5, 6]:

$$\varepsilon = \frac{\Delta\tau}{\ln \dfrac{\Delta P}{\Delta P - \sigma\Delta t}} . \quad (19)$$

We define the sensitivity η of the gauge as the ratio of the reactions $(t_1 - t_2)$ of the RC gauge to the measured heat flux P,

$$\eta = \frac{t_1 - t_2}{P} . \quad (20)$$

From Eq. (12) we obtain an expression for $(t_1 - t_2) = f(\tau)$, and instead of m_0 and R we take their values from Eq. (13):

$$t_1 - t_2 = \frac{1}{\sigma}\left[P(\tau) - C\frac{dt_1}{d\tau}\right]$$

and substitute into Eq. (19):

$$\eta = \frac{1}{\sigma}\left[1 - \frac{C}{P(\tau)} \cdot \frac{dt_1}{d\tau}\right] . \quad (21)$$

It is clear from Eq. (21) that the sensitivity, in general, depends on the structural parameters σ and C and the regime factors $P(\tau)$ and $dt_1/d\tau$. From analysis of the heat conditions for bodies 1 and 2 which make up the RC gauge, [4] obtained a dependence for $dt_1/d\tau$ for different regimes of variation of $P(\tau)$. These relations allow us to use Eq. (20) to determine the sensitivity in different measurement cases; in particular, for $dt_1/d\tau = 0$, i.e., under steady conditions, we obtain the well-known relation $\eta = \sigma^{-1}$.

In conclusion, we note that the present paper has considered only the theoretical possibilities of measuring a heat flux which varies with time using a self-contained instrument, a combination or an RC-type gauge. A good deal of analytical, experimental, and engineering work is required prior to practical application of this kind of instrument, capable of the required level of error, responsiveness, and sensitivity.

NOTATION

P, heat flux to be measured; g, k, coefficients of proportionality; ΔE, Δt_T, difference in potentials and temperatures at the surfaces of the ordinary heat gauge; τ, time; x, ambient coordinate; l, l_1, l_2, linear dimensions of the heat gauge; c, C, λ, ρ, specific and total heat capacities, thermal conductivity, and density of the ordinary heat gauge; S, surface area of the gauge; I[f], an operator; t_V, average volume temperature of the ordinary gauge; λ_T, t_T, thermal conductivity and temperature of a solid body; t_1, temperature at the surface ($x = l$) of the ordinary gauge; t_m, temperature of the medium surrounding the gauge; t_s, temperature at the surface ($x = l_1$) of the combination gauge; σ, R, thermal conductivity and resistance of the gap; Δt, temperature drop at the gap; t_1, t_2, temperatures of bodies 1 and 2 of the RC gauge; ψ, criterion for nonuniformity of temperature field in the body shown in Fig. 2b; $\Delta P/P$, error in determining the heat flux; $\Delta\tau$, time interval in which one must measure the minimum change in power ΔP; Δt_1, minimum change in temperature of body 1; η, sensitivity of the heat gauge; α, total heat-transfer coefficient; ε, thermal inertia coefficient.

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